Province of the
EASTERN CAPE
EDUCATION

SIT/ file

NATIONAL SENIOR CERTIFICATE

GRADE 12

SEPTEMBER 2023

MATHEMATICS P2

MARKS: 150

TIME: 3 hours

MATHE2

This question paper consists of 13 pages, including a 1-page information sheet, and an answer book of 25 pages.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs, etc. which you have used in determining the answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round off answers to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. An information sheet with formulae is included at the end of the question paper.
9. Write neatly and legibly.

QUESTION 1

1.1 A school's hockey team recorded the number of push-ups each player completed in a minute. The numbers for the seven players were:

29 27 24 31 22 19 30

1.1.1 Calculate the:

(a) Mean (2)

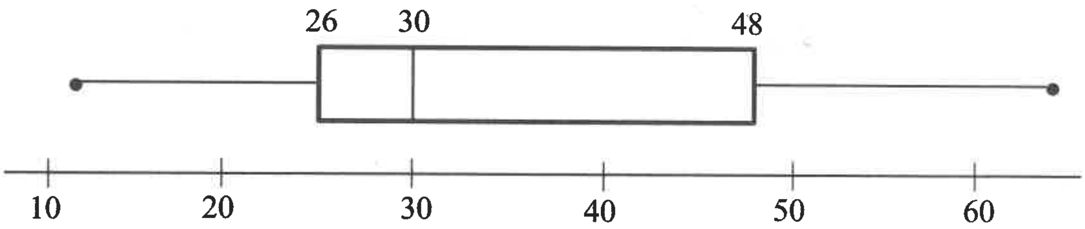
(b) Standard deviation (1)

1.1.2 How many players were within one deviation of the mean? (3)

1.1.3 Seven players in the school's rugby team also recorded the number of push-ups they completed in a minute. Their numbers gave a mean of 26 and a standard deviation of 3,2.

Use the standard deviations and the means to compare the number of push-ups of the players in the rugby and hockey teams. (2)

1.2 The number of points scored by a rugby team in each of 10 matches is represented in the box and whisker diagram below. The scores of the 10 matches were different.



1.2.1 In what percentage of the matches did the team score over 30 points? (1)

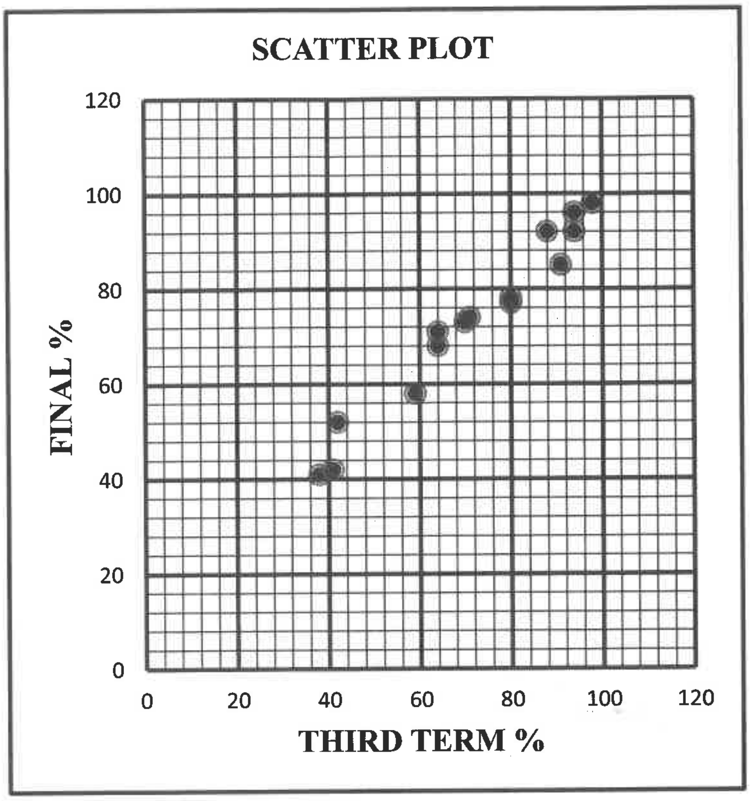
1.2.2 Which of the mean or median is likely to be greater? Give a reason for your answer. (2)

[11]

QUESTION 2

The table shows the percentages scored by a sample of 15 candidates in the third term and final examinations of 2022. The table and the scatter plot below represent these marks.

Third	71	80	59	38	41	98	80	88	91	94	64	94	70	42	64
Final	74	77	58	41	42	98	78	92	85	92	68	96	73	52	71



2.1 Determine the equation of the least squares regression line for the data, rounding off your answers to 3 decimal places. (3)

2.2 Write down the value of the correlation coefficient, r, between the 3rd term and final exam percentages. (1)

2.3 A candidate scored 48% in the third term.

2.3.1 Use the equation of the least squares regression line to predict his final percentage. Round your answer off to the nearest whole number. (2)

2.3.2 Give a reason why the prediction can be regarded as reliable. (1)

2.4 The least squares regression line is used to predict that the final percentage of a candidate who scored 50% in the third term is 80%.

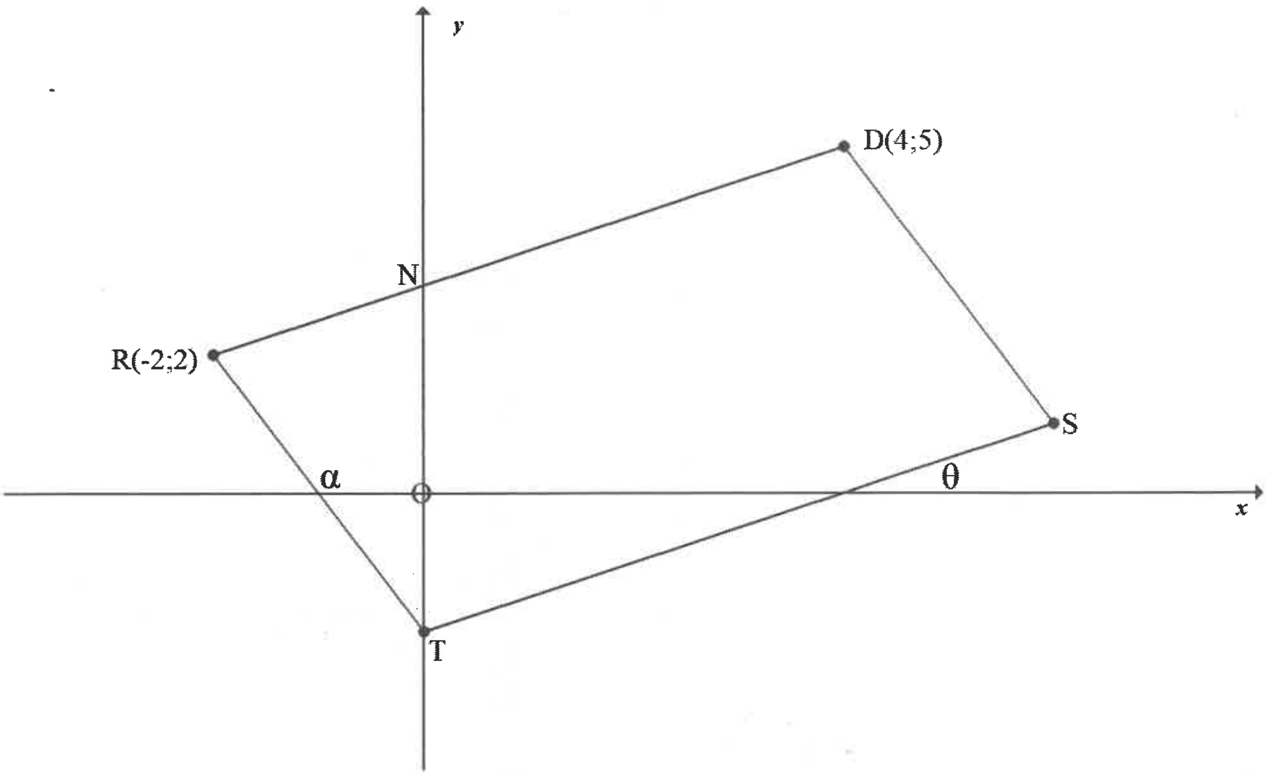
2.4.1 Why would this prediction be unreliable? (1)

2.4.2 Would adding the point (20;10) to the original data set increase or decrease the gradient of the least squares regression line? (1)

[9]

QUESTION 3

In the diagram below, D(4; 5), R(-2; 2), T and S form a quadrilateral. RD cuts the y-axis at N and T is a point on the y-axis. The inclinations of RT and TS are α and θ respectively. RD∥TS and the equation of TS is $y = \frac{1}{2}x - 2$.



3.1 Write down the coordinates of T. (1)

3.2 Calculate:

3.2.1 The gradient of RT (2)

3.2.2 The size of $R\hat{T}S$ (5)

3.3 Determine the equation of RD in the form $y = mx + c$. (3)

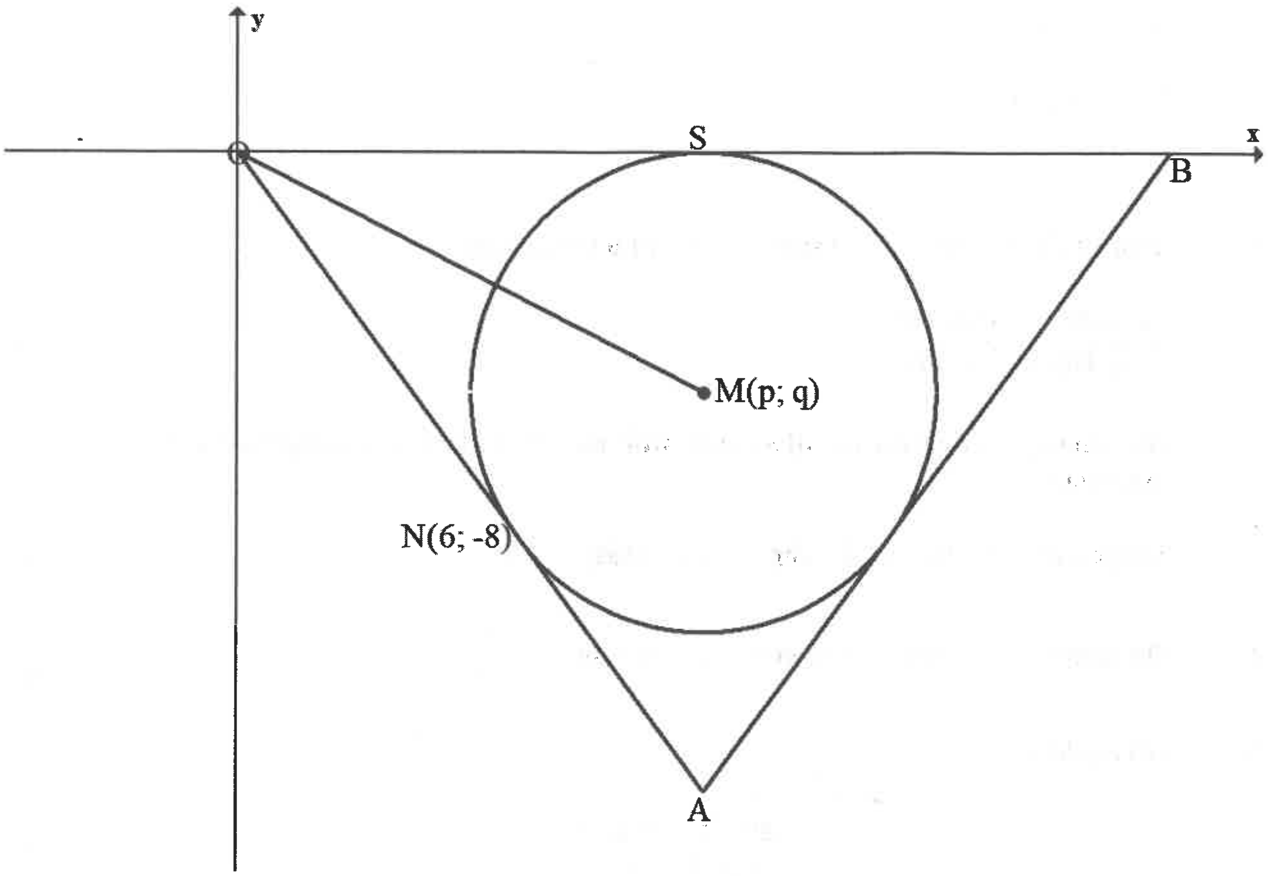
3.4 If RT∥DS, calculate the coordinates of M, the midpoint of RS. (3)

3.5 Calculate the area of ΔRTN. (4)

[18]

QUESTION 4

In the diagram below, a circle, centered at $M(p;q)$, touches the x-axis at S and the line OA is a tangent to the circle at $N(6;-8)$.



4.1 Calculate the:

4.1.1 Length of ON (2)

4.1.2 The value of p (2)

4.1.3 The gradient of NM (3)

4.1.4 The value of q (2)

4.2 Determine the equation of the circle in the form: $(x-a)^2+(y-b)^2=r^2$. (3)

4.3 $x=k$ is a tangent to the circle. Write down the value(s) of k. (2)

4.4 The line $y=-\frac{4}{3}x+t$ cuts the circle at two different points. Determine the values of t. (6)

4.5 Another circle with equation $(x-10)^2+(y-6)^2=25$ is given.

Will the two circles touch, cut or not? Give a reason for your answer. (2)

[22]

QUESTION 5

5.1 If $\sin 54° = p$, express each of the following in terms of p, **without the use of a calculator.**

5.1.1 $\sin 594°$ (2)

5.1.2 $\cos 36°$ (2)

5.1.3 $\cos 18°$ (4)

5.2 Simplify the following **without the use of a calculator**.

$$\frac{\cos 140° - \sin(90 - \theta)}{\sin 410° + \cos(-\theta)}$$ (6)

5.3 Determine, **without the use of a calculator**, the value of the following trigonometric expression.

$$\cos(x + 65°).\cos(x + 20°) - \sin(x + 245°).\sin(x + 20°)$$ (4)

5.4 Determine the general solution of: $\cos^2 x - \sin^2 x = \frac{1}{2}$ (4)

5.5 Given the identity:

$$\frac{\sin 2\theta.\tan\theta}{\cos 2\theta + 1} = \tan^2\theta$$

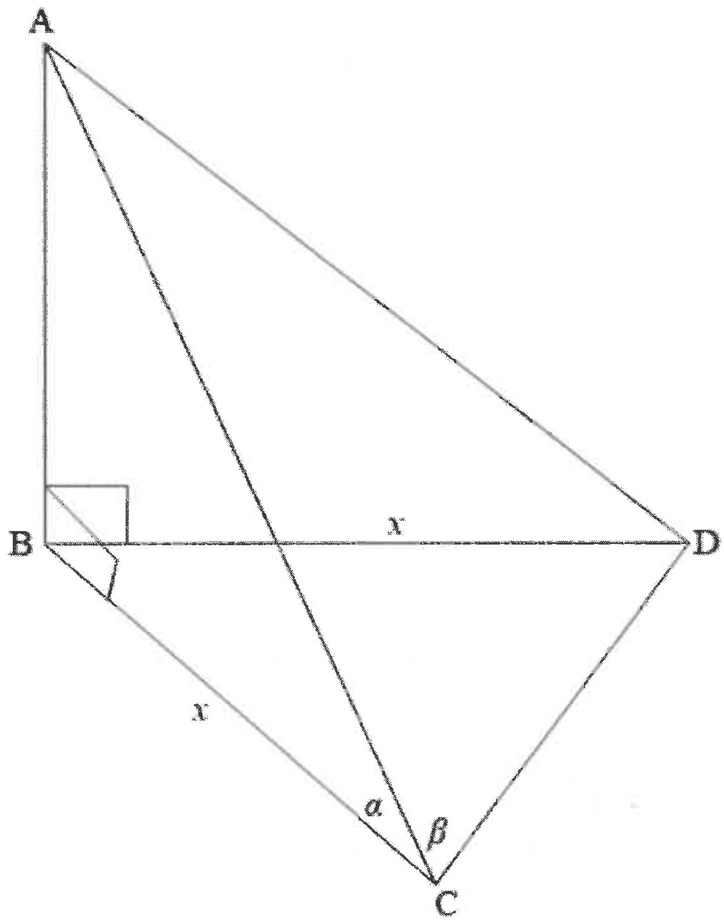
5.5.1 Prove the identity (4)

5.5.2 Determine the values of θ for which the identity is undefined if $0° \leq \theta \leq 180°$. (4)

[30]

QUESTION 6

In the figure below, B, C and D are points on the same horizontal plane. AB is a vertical tower with the angle of elevation from C to A equal to α and $A\hat{C}D = \beta$. $BD = BC = x$.



6.1 Why is AC = AD? (1)

6.2 Write AC in terms of x and α. (2)

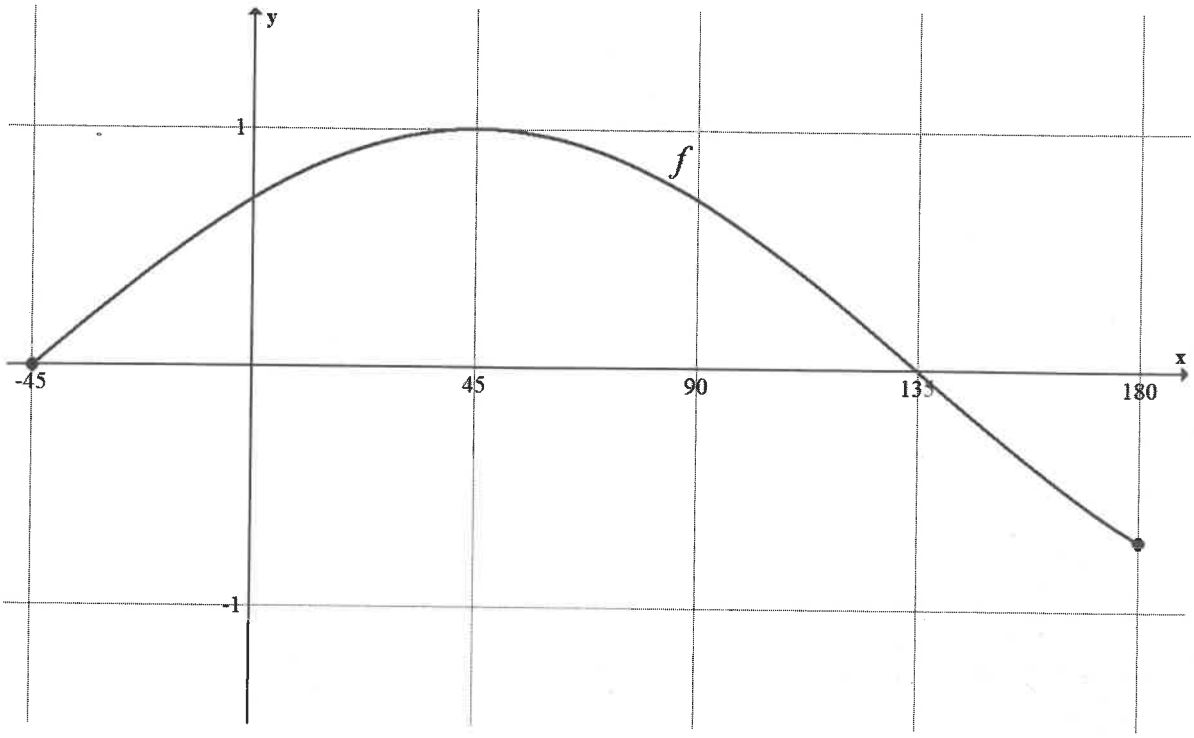
6.3 Show that $CD = \frac{2x \cos \beta}{\cos \alpha}$ (4)

6.4 Hence, determine the length of CD if $x = 25\ cm, \alpha = 30°$ and $\beta = 65,62°$. (2)

[9]

QUESTION 7

Sketched below is a graph of $f(x) = \cos(x - 45°)$ for $-45° \leq x \leq 180°$. Use the graph to answer the questions that follow.



7.1 Write down the range of f, for the given interval. (2)

7.2 Draw the graph of $h(x) = \sin 2x$, for $x \in [-45°;180°]$ on the same set of axes as f in the ANSWER BOOK. Indicate the coordinates of all intercepts with the axes as well as turning points. (3)

7.3 State the period of h. (1)

7.4 Use your graph to determine the values of x for which f and h are both increasing. (2)

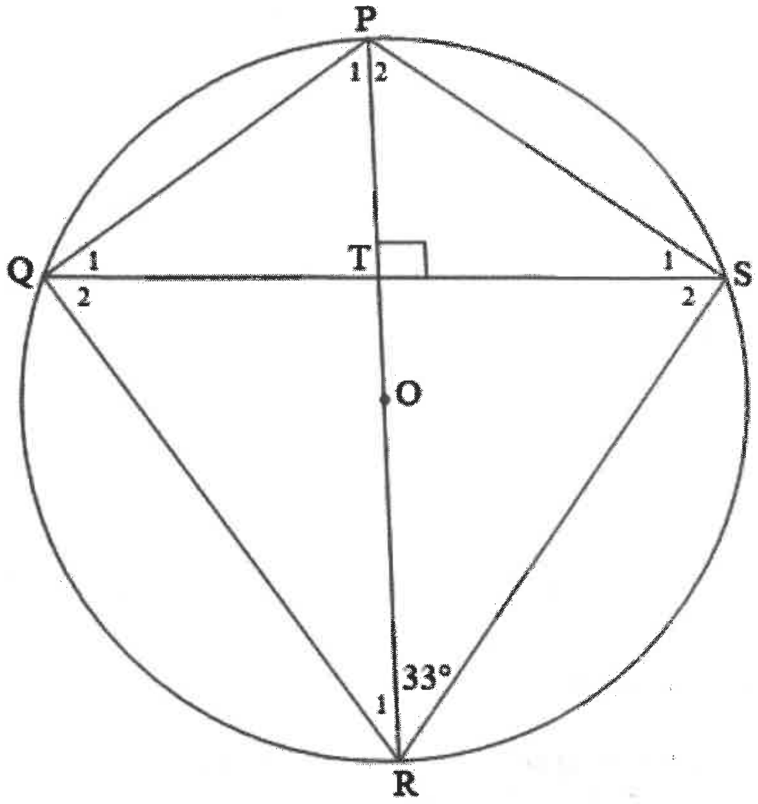
7.5 Determine the values of x for which $f(x) - h(x) = 1$. (2)

7.6 The graph of f is translated 60° to the left to form the graph of g. Write down the equation of g in the form: $g(x) =$ ____. (1)

[11]

QUESTION 8

In the diagram below, PR is a diameter of circle PQRS with centre O. PR intersects with chord QS at T such that $P\hat{T}S = 90°$. $P\hat{R}S = 33°$.



8.1 Determine, with reasons, the size of:

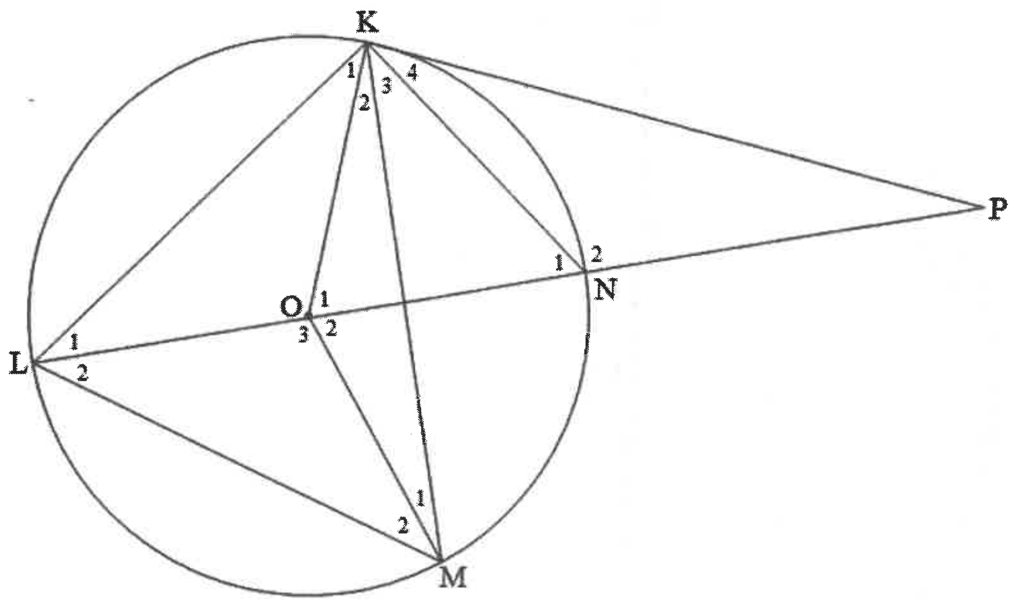
8.1.1 $\hat{P}_1$ (3)

8.1.2 $\hat{Q}_2$ (2)

8.2 If QS = 16 cm and PR = 20 cm, determine, with reasons, the length of TO. (4)
[9]

QUESTION 9

In the diagram below, O is the centre of the circle and KP is a tangent to the circle. LN, the diameter of the circle, is extended to meet KP at P. Straight lines OK, OM, KM and KN are drawn.



9.1 Write down two angles equal to $90°$. (2)

9.2 If $\hat{K}_4 = x$, write down the following angles in terms of x, giving reasons.

9.2.1 $\hat{L}_1$ (2)

9.2.2 $\hat{K}_1$ (2)

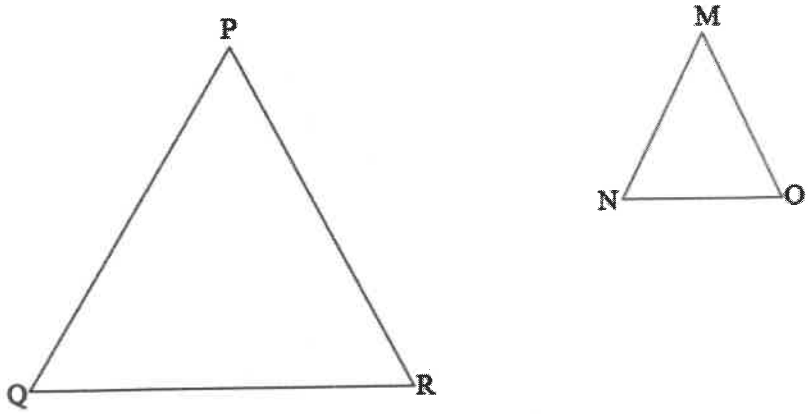
9.2.3 $\hat{P}$ (2)

9.3 Join MP, which is a tangent to the circle, and hence prove that KOMP is a cyclic quadrilateral. (3)

[11]

QUESTION 10

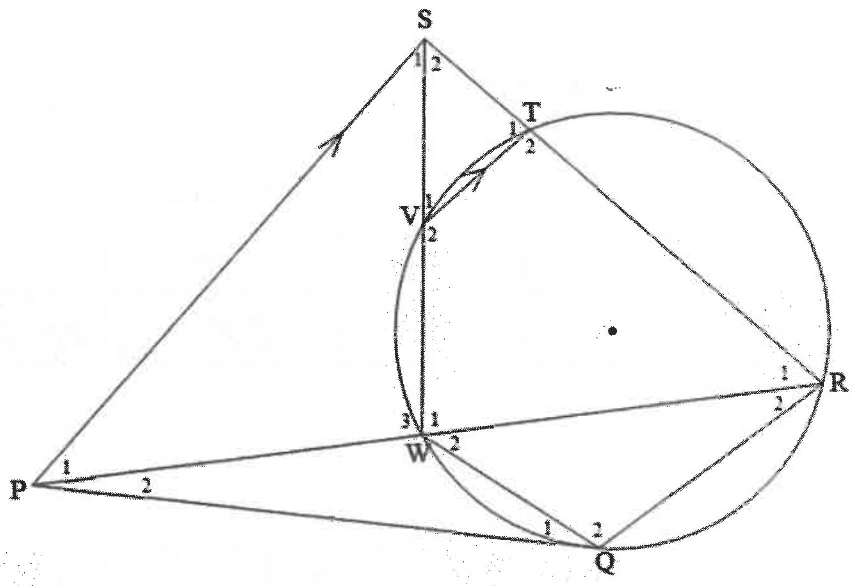
10.1 In the diagram below, ΔPQR and ΔMNO are given with $\hat{P} = \hat{M}$, $\hat{Q} = \hat{N}$ and $\hat{R} = \hat{O}$.



Use the diagram in your answer book to prove the theorem which states that:

$$\frac{MN}{PQ} = \frac{MO}{PR}$$ (6)

10.2 In the diagram below, PQ is a tangent to the circle at Q. R is a point on the circle and S lies outside the circle. PR cuts the circle in W and RS cuts the circle in T. SW cuts the circle in V. VT ∥ PS.



Prove that:

10.2.1 $\hat{S}_1 = \hat{R}_1$ (3)

10.2.2 $\Delta PWS\ |||\ \Delta PSR$ (3)

10.2.3 $PQ^2 = PW.PR$ (5)

10.2.4 $PQ = PS$ (3)

[20]

TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad A = P(1-ni)$ $\quad A = P(1-i)^n$ $\quad A = P(1+i)^n$

$\sum_{i=1}^{n} 1 = n$ $\quad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ $\quad T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}(2a + (n-1)d)$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a(r^n - 1)}{r-1}$; $r \neq 1$ $\quad S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $\quad P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad \mathrm{M}\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad a^2 = b^2 + c^2 - 2bc.\cos A$ $\quad area\,\Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\quad \sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum fx}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$